

Master QFin, Mathematical Finance  
Midterm Exam Tuesday, 13 May 2025

- Please mark every sheet with your name and Mat.Nr.
- Good luck !

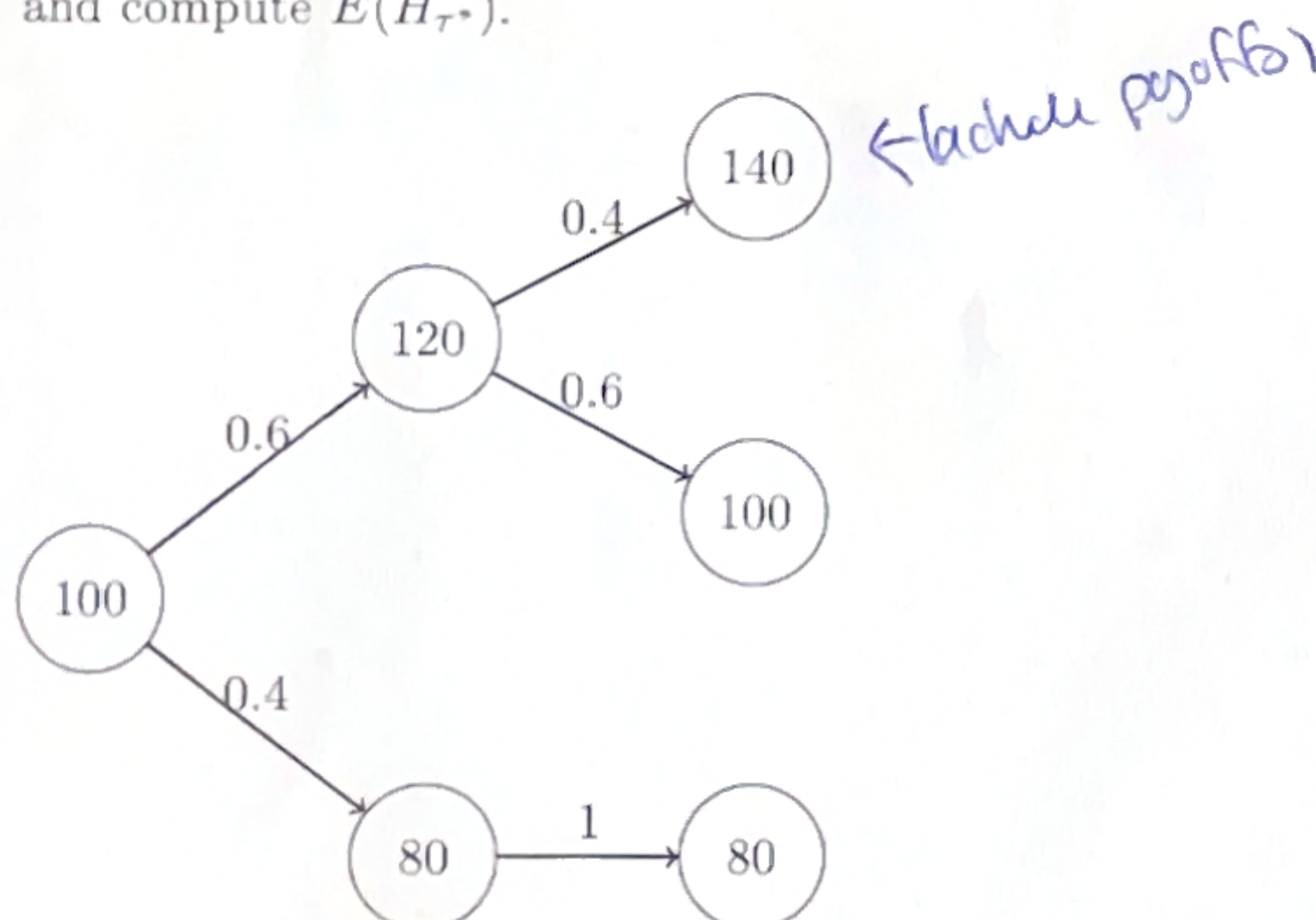
1. **Stopping times.** (4 points) Consider a filtered probability space  $(\Omega, \mathcal{F}, P)$  with filtration  $\{\mathcal{F}_n\}_{n=0,1,\dots}$ .

- a) (1 point) State the definition of a stopping time  $\tau$ .
- b) (3 points) Consider a firm with asset value given by an adapted process  $V_n$ ,  $n \in \mathbb{N}$ . Suppose that the firm defaults at the first time the asset value lies below a threshold  $K$  (which may be interpreted as liabilities of the firm), and denote the default time by

$$\tau = \inf\{n \in \mathbb{N} : V_n \leq K\}.$$

Show that  $\tau$  is a stopping time.

2. **Optimal stopping.** (4 points) A company has the option to enter into a project at times 0, 1 and 2. The profits  $H_n$  of entering at time  $n$  are given in the tree below, together with the probabilities of the different branches. Find the optimal time to enter into the project, that is the stopping time  $\tau^*$  that maximizes  $E(H_\tau)$  over all stopping times with values in  $n = 0, 1, 2$  and compute  $E(H_{\tau^*})$ .



3. **Discrete Financial Mathematics** (7 points) Consider a financial market model  $\mathcal{M}$  with two assets, the numeraire  $S_0$  and a risky asset  $S_1$ .

- a) (2 points) State the first and the second fundamental theorem of asset pricing for  $\mathcal{M}$ .
- b) (1 point) Give the definition of an admissible and selffinancing trading strategy  $\theta$  for  $\mathcal{M}$ .
- c) (4 points) Consider an admissible selffinancing trading strategy  $\theta$  for  $\mathcal{M}$  and suppose that  $\mathcal{M}$  admits an equivalent martingale measure  $Q$ . Define the gains from trade with respect to the discounted assets  $\tilde{G}_t^\theta$  and show that  $\tilde{G}_t^\theta$  is a  $Q$  martingale.